MANAGERIAL AND DECISION ECONOMICS, VOL. 13, 389–397 (1992)

Price Uncertainty and the Effect of Capital Costs in a Point in–Point out Inventory Investment

Anders Thorstenson
and
Peter Hultman
Linköping Institute of Technology, Sweden

This paper analyzes a point in–point out inventory investment under price uncertainty. The optimal quantity is determined by maximizing the expected value of the investor's risk preference function, which is a function of profit. Using an exponential risk preference function, the adjustment in the optimal quantity stemming from a change in the interest rate is investigated. The main conclusion is that the sign of the adjustment depends both on how profit is expressed and on the type of price distribution applied. Contrary to what is assumed in conventional managerial control practices, a rise in the interest rate might lead to an increase in the optimal quantity when present value serves as a measure of profit.

INTRODUCTION

There is a large number of studies in the literature on the subject of how the competitive firm's optimal output is affected by uncertainty in the selling price. The expected-utility-maximizing firm's attitude towards risk has been found to be decisive when comparing the optimal output under different circumstances. It has also been shown that the impact of changes in parameters (such as expected value and riskiness of the stochastic selling price, fixed production costs and rate of taxation) is dependent on the type of risk behavior that the firm follows. These issues have been studied by, among others, Sandmo (1971) and Leland (1972). Contributions have also been made by e.g. McCall (1967), Rothschild and Stiglitz (1971), Batra and Ullah (1974), and Ishii (1977). Further references are found in a comprehensive review of the problem by Lippman and McCall (1981).

The question posed in this paper is: How is the competitive firm's optimal quantity under price uncertainty affected by changes in capital cost? Our purpose is to derive a partial answer in the form of results for the type of problem referred to above when interpreted as a simple buy/store/sell situation.

In industry it is a common practice to determine ordering quantities as a trade-off between, on the one hand, the costs associated with ordering and, on the other, the costs of holding inventory. Larger ordering quantities means higher inventory levels and more capital tied up. In this framework an increase in the cost of capital will lead to lower ordering quantites (and lower inventory levels), and this relationship is frequently utilized by the management in companies. By changing the cost of capital used for procurement or production decisions, managers on lower levels are supposed to react correspondingly, which will lead to a desired change in average inventory level. The question that arises is: Can a company or a manager with a certain attitude towards risk and with a perceived uncertainty in a key parameter always be expected to react in this way?

In the same type of model studied here it was shown in Hultman and Thorstenson (1987) that *risk aversion* is not a sufficient condition for a decision maker to decrease the ordering quantity when the cost of capital increases. When profit is expressed as revenue minus cost, a company or a manager with *non-increasing absolute risk aversion* will always decrease the ordering quantity in response to an increase in the cost of capital. In this paper the aim

0143–6570/92/050389–09$09.50

is to investigate whether there are cases in which a decision maker who evaluates profit by the *net present value* method and whose risk attitude is described by non-increasing absolute risk aversion will increase ordering quantities when the cost of capital increases. The impact of the probability distribution on the ordering-quantity adjustment is also within the scope of this study.

THE MODEL

In the point in–point out inventory-model treated here we consider a one-period setting, in which a good is procured at the beginning of the period. Selling takes place at the end of the period in a competitive market, where the inventory investor's total supply of the good is absorbed by the market at the prevailing price. Thus, a critical element of the model is a fixed time lag between procurement and sale, which induces costs for capital tied up in inventory. This type of model might be appropriate, for example, when inventories are held for speculative purposes between certain points in time, or when the good has to undergo a process of maturity before it can be sold. It might also be applicable when the good has to be kept in inventory for a certain period awaiting the single season during which it is to be sold. To summarize: typical of this model is that the market is closed or nonexisting during a fixed time period. The investor's problem is then to decide, at a point in time when the selling price is unknown, the quantity of the good to be procured, stored and finally sold.

The end-of-period selling price is uncertain but has a known (subjective) probability distribution. Hence, the uncertainty in the model stems from the selling price and, as a consequence, the revenues are stochastic. The cost function, however, is known with certainty. In general, there are three kinds of costs to be considered in inventory models: procurement or production cost, fixed ordering or set-up cost, and inventory holding cost. In the present model the cost function includes all these costs, fixed as well as variable, except the part of the holding cost attributable to the cost of capital tied up in inventory. The reason for excluding capital costs is to facilitate an explicit treatment of their effect on the optimal quantity. A fixed ordering or set-up cost can, without loss of generality, be incorporated into the cost function if it is assumed that procurement always will take place. The part of the holding cost included in the cost function represents costs for physical handling, storage space, insurance, etc. These costs are also a known function of the ordering quantity, and, for ease of exposition, are treated as an advance lease payment.

When determining his optimal quantity, the investor is assumed to seek to maximize the expected value of his von Neumann–Morgenstern risk preference function, whose argument is the profit from the one-period inventory holding. For comparative purposes, two types of expressions for profit are used. On the one hand, profit is defined in a traditional (accounting) manner as revenue minus cost including capital cost. On the other, profit is the net present value of the in- and out-payments associated with the inventory investment. The basis for (and effects of) using these two principles in inventory models have been studied by Thorstenson (1988). The difference between the profit expressions is due to the principle determining capital costs. In the traditional case capital cost is expressed as the interest rate multiplied by capital tied up in inventory. In the net present value case all costs and revenues, including capital cost, are determined from the net present value of the payments.

ANALYSIS

Let π denote the profit of the inventory investment and let subscripts T and N denote the traditional (revenue minus cost) case and the net present value case, respectively. Furthermore, let $c(Q)$ be the sum of costs for procurement, ordering, and holding (except costs for capital tied up), as a function of the ordering quantity Q, and let marginal costs $c'(Q)$ be positive. The future selling price p is assumed to be uncertain with an assessed probability distribution, expected value $\bar{p}$ and variance σ_p^2. In addition to the cost $c(Q)$, capital costs are considered via the effect of the discount factor R^{-1} $(R>1)$. The two profit expressions can now be written as

$$\pi_T = pQ - c(Q)R \tag{1}$$

$$\pi_N = pQR^{-1} - c(Q) \tag{2}$$

The discount factor R^{-1} is based on the risk-free interest rate, since uncertainty is accounted for by considering the investor's risk preference function V. V is assumed to be twice differentiable and monotonically increasing $(V'>0)$. Unless otherwise stated, we also assume that it is strictly concave $(V''<0)$.

If the risk preference function is taken to be a function of profit, $V(\pi)$, the investor's problem is formulated as

$$\underset{Q}{\text{Max}}\ E[V(\pi)] \tag{3}$$

where E is the expectations operator. The first- and second-order conditions are then given by

$$E\left[V'(\pi)\frac{\partial\pi}{\partial Q}\right]=0 \tag{4}$$

$$E\left[V''(\pi)\left(\frac{\partial\pi}{\partial Q}\right)^2+V'(\pi)\frac{\partial^2\pi}{\partial Q^2}\right]<0 \tag{5}$$

where $V'(\pi)$ and $V''(\pi)$ represent the first- and second-order derivative of the risk preference function with respect to profit. It is assumed that these conditions hold in such a way that a positive and unique solution Q^* exists.

For the derivative of profit π with respect to the quantity Q we have

$$\frac{\partial\pi_T}{\partial Q}=p-c'(Q)R \tag{6}$$

$$\frac{\partial\pi_N}{\partial Q}=pR^{-1}-c'(Q) \tag{7}$$

The first-order condition can be developed into

$$E\left[\frac{\partial\pi}{\partial Q}\right]=-\frac{\text{Cov}[V'(\pi),\partial\pi/\partial Q]}{E[V'(\pi)]} \tag{8}$$

where $\text{Cov}[\cdot,\cdot]$ denotes the covariance between the two arguments.

Hence, it follows that in the special case when price is certain and equal to $\bar{p}$, say, or when the investor is risk neutral, i.e. $V'(\pi)$ is constant, the covariance is zero and the solution is given by

$$\bar{p}-c'(Q)R=0 \tag{9}$$

This result is well known from the microeconomic theory of the firm under certainty, and holds irrespective of which of the two profit expressions is used.

If, however, the investor is risk averse, ($V''(\pi)<0$), it can be shown (see e.g. Ishii, 1977; Rothschild and Stiglitz, 1971) that the covariance is negative and hence that $E[\partial\pi/\partial Q]$ is positive. Therefore, the optimal quantity in the case of a risk-averse investor is always smaller (if $c''(Q)>0$) than in the case when the investor is risk neutral. This result is also given in Sandmo (1971).

Alternatively, the first-order condition can be expressed as (see also Rothschild and Stiglitz, 1971)

$$\frac{E[V'(\pi)p]}{E[V'(\pi)]}-c'(Q)R=0 \tag{10}$$

By developing the expected values in Eqn (10) for a particular risk preference function and for a certain price distribution, the effect of a change in the interest rate can be treated explicitly.

In the examples that follow we shall assume that the risk preference function is the (negative) exponential

$$V(\pi)=-e^{-\phi(\pi+w)} \tag{11}$$

where ϕ is a positive constant and w represents the investor's initial endowment. This risk preference function is commonly applied in the literature, and according to Hammond (1974) it is often a good approximation to other, less tractible, preference functions. The exponential risk preference function is monotonically increasing and strictly concave, thereby representing risk-averse behaviour. Furthermore, the degree of absolute risk aversion is constant ($=\phi$), i.e. non-increasing in the Arrow–Pratt sense (see e.g. Pratt, 1964). Hence, the initial endowment, w, does not affect the analysis and is therefore omitted in the following.

From the chosen risk preference function it follows that $V'(\pi)=Ae^{Kp}$, with A and K being constants, for each case determined by the profit function. Taking the expectation yields

$$E[V'(\pi)]=AE[e^{Kp}]=Am(K) \tag{12}$$

with $m(K)$ being analogous to the *moment-generating function* of the probability distribution of p. In the case of a non-negative random p, it corresponds to the Laplace transform of the distribution of p (see Feller, 1966, pp. 410–11). This result is characteristic of the exponential risk preference function, as has also been observed by McCall (1967). From this observation it follows that

$$E[V'(\pi)p]=AE[pe^{Kp}]=A\frac{dE[e^{Kp}]}{dK}$$
$$=A\frac{dm(K)}{dK} \tag{13}$$

Combining Eqns (12) and (13) gives

$$\frac{E[V'(\pi)p]}{E[V'(\pi)]}=\frac{dm(K)/dK}{m(K)}$$
$$=\frac{d(\log m(K))}{dK}=z(K) \tag{14}$$

where $z(K)$ is introduced to simplify the notation.

In order to obtain an expression of changes in the optimal quantity with respect to changes in the discount factor we apply the usual comparative statics approach. The first-order condition is therefore differentiated according to

$$\frac{\partial^2 E[V(\pi)]}{\partial Q^2} dQ + \frac{\partial^2 E[V(\pi)]}{\partial Q \partial R} dR = 0 \qquad (15)$$

which implies that

$$\frac{\partial Q}{\partial R} = -\frac{\partial^2 E[V(\pi)]/\partial Q \partial R}{\partial^2 E[V(\pi)]/\partial Q^2} \qquad (16)$$

Since the denominator in the right hand side of Eqn (16) is strictly negative, according to the second-order condition, we have that

$$\text{Sign}\,\frac{\partial Q}{\partial R} = \text{Sign}\,\frac{\partial^2 E[V(\pi)]}{\partial Q \partial R} \qquad (17)$$

From the above (Eqns (10) and (14)) we know that at the optimum

$$\text{Sign}\,\frac{\partial^2 E[V(\pi)]}{\partial Q \partial R} = \text{Sign}\,\frac{\partial}{\partial R}[z(K) - c'(Q)R] \qquad (18)$$

and combining this with Eqn (17) leads to

$$\text{Sign}\,\frac{\partial Q}{\partial R} = \text{Sign}\left[\frac{\partial z(K)}{\partial R} - c'(Q)\right] \qquad (19)$$

In the traditional case, identification of K yields $K = -\phi Q$, and since here K is not a function of the discount factor R^{-1}, nor is $z(K)$, the sign of $\partial Q/\partial R$ will therefore be negative. From this we can conclude that, in the case of the traditional profit expression, an increase in the interest rate always leads to a decrease of the optimal quantity, irrespective of the type of probability distribution the selling price follows. This result agrees with the intuitive notion of the effect of increased capital costs.

Turning to the profit expression based on net present value, identification of K yields $K = -\phi Q R^{-1}$, and since K in this case is a function of R, nothing can be said of the sign of $\partial Q/\partial R$ without further investigation. From Eqns (10) and (14) we find that at the optimum

$$c'(Q) = z(K) R^{-1} \qquad (20)$$

and since

$$\frac{\partial z(K)}{\partial R} = \frac{\partial z(K)}{\partial K}\frac{\partial K}{\partial R} = -R^{-1} K \frac{\partial z(K)}{\partial K} \qquad (21)$$

it follows that

$$\text{Sign}\,\frac{\partial Q}{\partial R} = \text{Sign}\left[-R^{-1}\frac{\partial}{\partial K}(Kz(K))\right] \qquad (22)$$

In Tables 1 and 2 the characteristics of some probability distributions are presented together with the resulting sign of $\partial Q/\partial R$ for each distribution. Results are given for the Gamma distribution (including three special cases), the Normal, the Poisson, and the Binomial. We are not suggesting that the distributions chosen are the most typical of price behavior in a setting like this. They are employed here simply in order to illustrate possible effects.

As can be seen from Table 2, a rise in the interest rate might lead to an increase in the optimal quantity ($\partial Q/\partial R > 0$), if price follows a Normal, a Poisson, or a Binomial distribution. This is the case if the variance of price and the degree of absolute risk aversion are sufficiently large. This somewhat counter-intuitive result will not occur when price follows a Gamma distribution, since an interest rate increase in those cases will always be followed by a decreased optimal quantity. This also includes the case when price is certain, as noted above. Hence, the sign of the adjustment in the optimal quantity as a result of a change in the interest rate depends on both the profit expression and the distribution of the price.

These results also confirm the analysis in Hultman and Thorstenson (1987), wherein non-increasing absolute risk aversion could not be shown to be sufficient for $\partial Q/\partial r < 0$ when profit was expressed using a present value approach. While this conclusion may appear unexpected, similar 'counter-intuitive' effects are also found in related models treated by e.g. Hite (1979) and Hartman (1972). In Hite's model, which is based on the CAPM (Capital Asset Pricing Model) framework, a rise in the risk-free interest rate might result in an increase in the optimal output. Hartman (1972) develops a model of a risk-neutral firm and examines the effect of increased uncertainty. He finds that the firm's investment might rise when uncertainty in future output prices is increased.

A partial explanation for our result when p and (consequently) π_N are normally distributed is the following. Since the risk preference function is exponential, different alternatives are ranked according to the measure λ_1, defined by

$$\begin{aligned}\lambda_1 &= E[\pi_N] - \gamma_1 \,\text{Var}[\pi_N] \\ &= \bar{p}QR^{-1} - c(Q) - \frac{\phi}{2}\sigma_p^2 Q^2 R^{-2} \qquad (23)\end{aligned}$$

Table 1. Density Function and Moment-Generating Function for the Gamma, Normal, Poisson, and Binomial Distributions

Probability distribution	Density function	Parameters	Exp. value ($\bar{p}$)	Variance (σ_p^2)	Moment-generating function ($m(K)$)
Gamma	$\frac{\beta^{\alpha} p^{\alpha-1} e^{-\beta p}}{\Gamma(\alpha)}$	$\alpha>0,\ \beta>0$	α/β	α/β^2	$\left(1-\frac{1}{\beta}K\right)^{-\alpha}$ [b]
— Erlang	see Gamma	$\alpha=n,\ \beta=n\theta$; n: integer	$1/\theta$	$1/n\theta^2$	$\left(1-\frac{1}{n\theta}K\right)^{-n}$
--- Exponential	see Gamma	$\alpha=1,\ \beta=\theta$	$1/\theta$	$1/\theta^2$	$\left(1-\frac{1}{\theta}K\right)^{-1}$
··· Constant	see Gamma	$\alpha=n\to\infty$; $\beta=n\theta$	$1/\theta$ [a]	0	$e^{K/\theta}$
Normal	$\frac{e^{-(p-\mu)^2/2\sigma^2}}{\sigma\sqrt{(2\pi)}}$	$\mu,\ \sigma$	μ	σ^2	$e^{\mu K+(\sigma^2K^2)/2}$
Poisson	$\frac{e^{-\lambda}\lambda^{p}}{p!}$; p: integer	λ	λ	λ	$e^{\lambda(e^K-1)}$
Binomial	$\binom{n}{p}\theta^{p}(1-\theta)^{n-p}$; p: integer	$0<\theta<1$; n: integer	$n\theta$	$n\theta(1-\theta)$	$(1+\theta(e^K-1))^n$

[a] Certain value.
[b] $K<\beta$.

where Var[·] denotes the variance and γ_1 is the constant $\phi/2$, reflecting the degree of risk aversion. With a Normal distribution and exponential preferences the measure λ_1 is the so-called *certainty equivalent* measure of profit.

Suppose that $Q_1<Q_2$. Then Q_1 is preferred to Q_2 if and only if

$$\bar{p}-\frac{c(Q_1)-c(Q_2)}{Q_1-Q_2}R-\frac{\phi}{2}\sigma_p^2(Q_1+Q_2)R^{-1}<0 \qquad (24)$$

An increase in R can change the sign of the left-hand side in Eqn (24) and can therefore reverse the order of preferences. Hence, a larger quantity is sometimes preferred when the interest rate is increased. Using π_T, this will never be the case. These are the same results as found in our comparative statics analysis above.

If, however, the tails of the Normal distribution do not reflect the investor's assessment of reality it might be useful to consider a truncated Normal as a description of price behavior. Norgaard and Killeen (1980) have demonstrated that with exponential risk preferences and a truncated Normal distribution the ranking is approximately given according to the measure λ_2 defined by

$$\lambda_2=E[\pi_N]-\gamma_2\sqrt{(\mathrm{Var}[\pi_N])} \qquad (25)$$

where γ_2 is a constant.

Using the same assumptions as above, the preference ordering in this case can never be reversed by an increase in R. Therefore, we conclude that the effect of the tails of the Normal distribution is (not unexpectedly) an important cause for the ambigous sign of the quantity adjustment. In particular, with risk aversion the effect of the lower tail is important. The ambigous sign of the quantity adjustment is also present with the Poisson and Binomial distributions, although the maximum loss is bounded for these two distributions.

AN EXAMPLE

Let us assume that a decision maker with a risk-preference function of the same type as above (see

Table 2. **Sign of Change in Optimal Ordering Quantity Due to Change in Interest Rate, when Applying Present Value-Based Profit Function ($K = -\phi Q R^{-1}$)**

Probability distribution	$z(K)$	$\left[-R^{-1}\frac{\partial}{\partial K}(Kz(K))\right]$	Sign $\frac{\partial Q}{\partial R}$
Gamma	$\frac{\alpha}{\beta - K}$	$\frac{-\bar{p}R}{(R+\phi Q\bar{p}/\alpha)^2}$	$-$
—Erlang	$\frac{1}{\theta - K/n}$	$\frac{-\bar{p}R}{(R+\phi Q\bar{p}/n)^2}$	$-$
—Exponential	$\frac{1}{\theta - K}$	$\frac{-\bar{p}R}{(R+\phi Q\bar{p})^2}$	$-$
—Constant	$1/\theta$	$\frac{-\bar{p}}{R}$	$-$
Normal	$\mu + \sigma^2 K$	$\frac{2\phi Q\sigma_p^2 - \bar{p}R}{R^2}$	$+/-$
Poisson	λe^K	$\frac{\bar{p}}{R}\left(\frac{\phi Q}{R} - 1\right)e^{-\phi Q R^{-1}}$	$+/-$
Binomial	$\frac{n\theta e^K}{[1+\theta(e^K - 1)]}$	$\frac{-z(K)}{R}\left[1 + \frac{K(1-\bar{p}/n)}{1+(e^K-1)\bar{p}/n}\right]$	$+/-$

Eqn (11)), i.e. with constant absolute risk aversion, uses a net present value expression for profit (as defined in Eqn (2)). This decision maker is facing a situation in which he has to decide upon the quantity (Q) of a good that is to be procured, stored for one time period, and then sold. Furthermore, suppose that the expected gross contribution margin ratio, $(\bar{p}Q - c(Q))/\bar{p}Q$, is constant and equal to a.

If the selling price one period ahead follows a Normal distribution, the optimal quantity, which can be found from Eqn (20), becomes

$$Q = \frac{vR}{\phi}[1 - (1-a)R] \tag{26}$$

where $v \equiv \bar{p}/\sigma_p^2$. The sign of $\partial Q/\partial R$ is determined by the size of a compared to that of R, and is in this case affected neither by the degree of risk aversion nor by the variance in the selling price. A positive sign is obtained if $a > 1 - R^{-1}/2$.

The size of the quantity adjustment ($\Delta Q = Q_2 - Q_1$), in absolute and relative terms, that will occur due to a finite change in the interest rate ($\Delta R = R_2 - R_1$) can also be found from Eqn (26):

$$\Delta Q = \frac{v(1-a)}{\phi}[\Delta R((1-a)^{-1} - 2R_1) - (\Delta R)^2] \tag{27}$$

$$\Delta Q/Q_1 = \frac{\Delta R}{R_1}\left[1 - \frac{\Delta R + R_1}{(1-a)^{-1} - R_1}\right] \tag{28}$$

Since $v > 0$, $\phi > 0$, and $0 < a < 1$, the sign of the quantity adjustment is determined by the expression inside the brackets in Eqn (27). It can also be observed that the relative change in optimal quantity is unrelated to the degree of risk aversion and the selling-price variance.

In Fig. 1 the quantity adjustment is shown as a function of a change in the interest rate when $a < 1 - R_1^{-1}/2$ (curve 1) and when $a > 1 - R_1^{-1}/2$ (curve 2). The counter-intuitive effect corresponds to those parts of the curves being in the first and third quadrants of the figure. As can be seen, a sufficiently large decrease ($\Delta R < A_1 = (1-a)^{-1} - 2R_1 < 0$) or increase ($\Delta R > A_2 = (1-a)^{-1} - 2R_1 > 0$) will always lead to a decrease in the optimal quantity ($\Delta Q < 0$), which in the first case (curve 1) is a counter-intuitive effect.

A special case occurs when $a = 1 - R_1^{-1}/2$, since the optimal quantity will decrease irrespective of

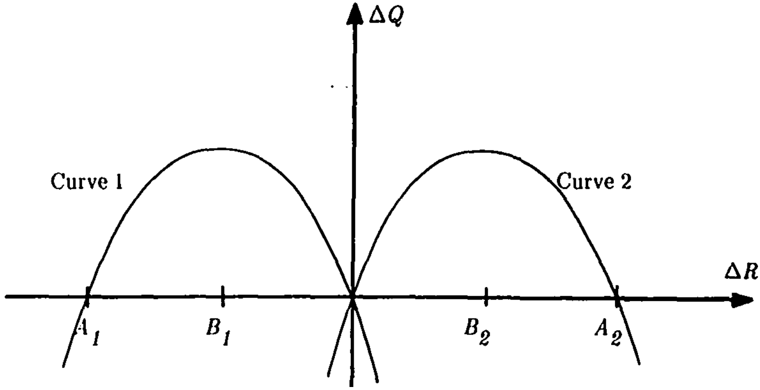

Figure 1. Change in optimal quantity due to change in interest rate when curve 1: $a<1-R_1^{-1}/2$; curve 2: $a>1-R_1^{-1}/2$.

the sign of ΔR. In the limiting case $R_1^{-1}=1$ (corresponding to a zero interest rate initially), the quantity adjustment will be positive when the interest rate is increased (corresponding to curve 2) if $a>0.5$. Thus, with a non-negative interest rate a curve 2 situation will not occur if the expected gross contribution margin ratio is less than 50%. If we assume that $R_1^{-1}=1$ and $a=0.75$, then ΔQ will have its maximum at $\Delta R=1$ ($=B_2$) and it will be negative when $\Delta R<0$ or $\Delta R>2$ ($=A_2$). From Eqn (28) we find that when $\Delta R=1$ the relative increase in Q is 1/3.

If $R_1^{-1}=0.95$ (corresponding to, for example, an annual interest rate of 5.3% and a one-year period) then an expected gross contribution margin ratio of at least 52.5% is required to obtain a curve 2 situation. With $a=0.75$ and $R_2^{-1}=0.90$ (i.e. $\Delta R=0.0585$), which must be considered as a less extreme change of interest rate than the previous case, the optimal quantity will increase by approximately 3.5%.

Evidently the size of typical market interest rate levels and fluctuations would yield a fairly small effect in this setting. If, however, the interest rate is used as a management control parameter for regulating inventory levels, as is often the case in practical inventory management, then a wider spectrum of interest rate levels is frequently applied, and, hence, the quantity adjustment effects would increase. A prerequisite for the counter-intuitive effect to appear is that the good involved has a sufficiently large expected contribution margin ratio. This means that the likelihood of finding examples of this effect in industry is largest in firms that calculate with relatively small direct or variable costs.

DISCUSSION

The basic issue with which this paper deals is the effect of capital cost when using two different expressions for profit as arguments in the investor's risk-preference function. The first of these we denoted the 'traditional' profit expression. The reason for this notation is the fact that traditional inventory models normally use costs incurred as the basis for calculating holding costs in general and capital costs in particular. In the simple inventory model treated here, capital cost is expressed using the interest on cost during the inventory holding period. The net present value-based profit expression, in contrast, associates capital cost with revenues, since future receipts are discounted to the present using the interest rate. Under certainty or risk neutrality either profit expression yields the same result in terms of the optimal solution. Under uncertainty and risk aversion, however, this is no longer true in general. Moreover, the sign of the adjustment in the optimal quantity resulting from a change in the interest rate may differ for the two profit expressions, even if non-increasing absolute risk aversion is assumed. In particular, this was shown by the examples in the previous section in which price followed a Normal, a Poisson, or a Binomial distribution.

The term 'terminal value'-based profit could equally well have been used to denote the traditional profit expression. This is the case, since π_T is equivalent to the terminal or future value of the inventory investment, i.e. the value at the time of sales. The issue here, then, is whether future or present value is the proper argument in the investor's risk-preference function. This issue (i.e. the order in which preference scaling and discounting are performed) has been analyzed by Scarsini (1986). He concludes that interchanging these operators results in different stochastic-dominance conditions for random cash flows. Our observations seem to be in accordance with Scarsini's result.

Another fundamental question in models of firm behavior concerns the choice of the firm's objective function. The problem of specifying what objective function exists for a firm is an important and yet not-settled issue in the finance literature. The model in this paper follows the approach used by Sandmo (1971) and Leland (1972) and others. This approach, implying that the firm acts to maximize the expected value of its risk preference function, has been criticized from at least two different standpoints (see e.g. Fama and Miller, 1972; Hite, 1979). First, it neglects the influence on valuation from the existence of a (perfect) capital market. If such a market exists, these influences should be reflected in the use of a market equilibrium model, which obviates the need for a risk-preference function for the firm. Second, in general it is impossible to determine an aggregate risk preference function for a firm guaranteeing unanimity among different owners. Hence, the use of such a function would imply that the firm is 'personified'.

As can be concluded from the discussion above, however, there are circumstances when maximizing expected utility could be an objective relevant to a firm. For example, this may be the case if important capital market imperfections prevail, such that assumptions underlying perfect market equilibrium models are no longer valid. Moreover, risk-preference functions for firms may be appropriate either in some special cases of owner preferences or where there is a single owner/manager or entrepreneur. They may also be appropriate when the firm's behavior is determined by a strong manager acting as an agent for the owners. In studies of possible dysfunctional behavior within a firm with decentralized operating units, the discussion of a manager's risk preferences may also be relevant to the firm.

In this paper we have made the assumption that a firm's or a manager's actions are influenced by risk-averse preferences. We have also assumed that the profit of an inventory investment might be assessed on the basis of either its present or its terminal value. Our analysis serves the purpose of showing that, even with non-increasing absolute risk aversion on behalf of the decision maker, an increase in the capital cost parameter cannot always be expected to induce a smaller ordering quantity. Since this conflicts with the conventional assumptions, our result is of importance to managerial control practices, because an interest rate change may not always give the intuitively expected effect on inventory levels.

As regards future research, a natural extension would be to consider risk-preference functions other than the negative exponential—for example, risk-preference functions with decreasing absolute risk aversion—or to analyze the dependence on relative risk aversion. Another kind of development of the model treated here would be to replace the fixed time horizon with a random time until the market opens. It would also be of interest to treat more general types of inventory models under uncertainty.

Acknowledgments

The authors are indebted to Robert W. Grubbström and two anonymous referees for valuable comments and suggestions.

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